

**Solution for the case**

**“Motorcycle Helmets with Bluetooth: Production”**

**1. A firm in a competitive industry has the cost curve  $C = \$1,000,000 + \$0.002q^2$ .  
The market price is  $P = 80$ . (All costs and prices refer to long-run values.)**

**a) Find fixed cost.**

Fixed Cost = \$1,000,000. (This is the amount we have to pay irrespective of the number of units produced. We can just take  $q=0$  and see what is the corresponding cost.)

**b) Find the marginal cost (MC).**

$$MC = C'(q) = \$0.004q.$$

**c) How much should the firm produce?**

First, let's find the number of units that would maximize the profit.

$$\begin{aligned}\text{Profit} = Pr(q) &= \text{Revenue} - \text{Cost} \\ &= P * q - (1,000,00 + 0.002q^2) \\ &= 80 q - 1,000,000 - 0.002q^2\end{aligned}$$

The first-order condition is

$$Pr' = 0, \text{ or } 80 - 0.004q. \text{ (Note that it is of the form } Pr' = P - MC = 0, \text{ so } P=MC.)$$

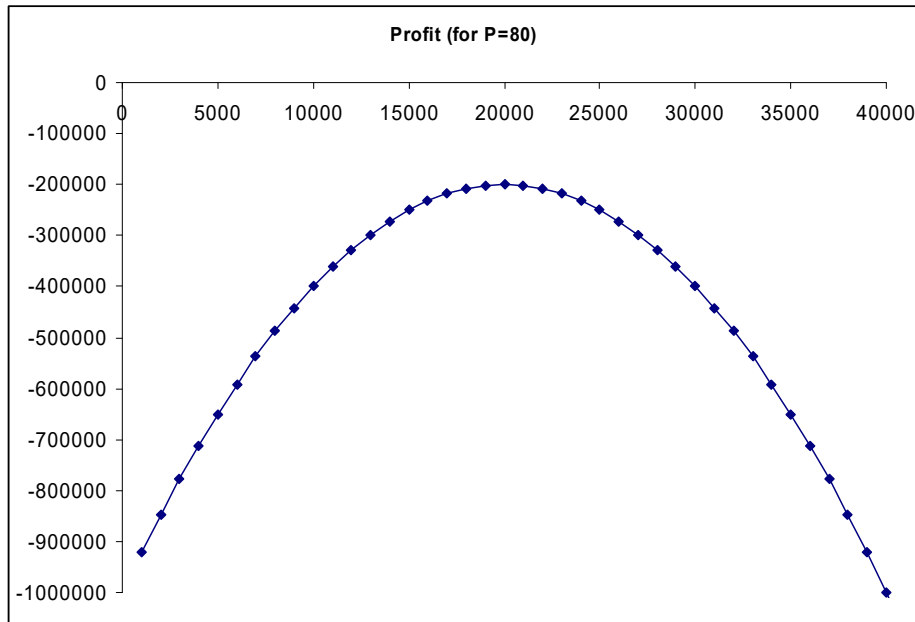
Equation for the optimal  $q$ :

$$80 = 0.004q, \text{ so } q = 80/0.004 = 20,000.$$

In other words, if we have to produce, we need to produce  $q=20,000$  units. However, the corresponding profit is:

$$\text{Corresponding profit} = 80 \times 20,000 - 1,000,000 - 0.002 \times 20,000^2 = -200,000 < 0.$$

Thus, for  $P = 80$ , profit is always negative.  $\Leftrightarrow$  The firm should shut down.



- d) Find the supply function for the firm.  
(i.e., produced quantity as a function of market price P)

$$\begin{aligned} \text{Profit} = \text{Pr}(q) &= \text{Revenue} - \text{Cost} \\ &= P * q - (1,000,000 + 0.002 q^2) \end{aligned}$$

Profit is maximized when

$$\text{Pr}'(q)=0$$

$$P - 0.004q=0$$

$$P = 0.004q$$

$$q = P/0.004=250 P$$

Corresponding value of Profit

$$\text{Pr} = P(250 P) - (1,000,000 + 0.002(250 P)^2)$$

$$\text{Pr} = (250 P)*(P-0.002*250 P) - 1,000,000 = 125 P^2 -1,000,000$$

Profit should be positive.

Profit is positive if  $125 P^2 -1,000,000 > 0$

$$P^2 > 8,000$$

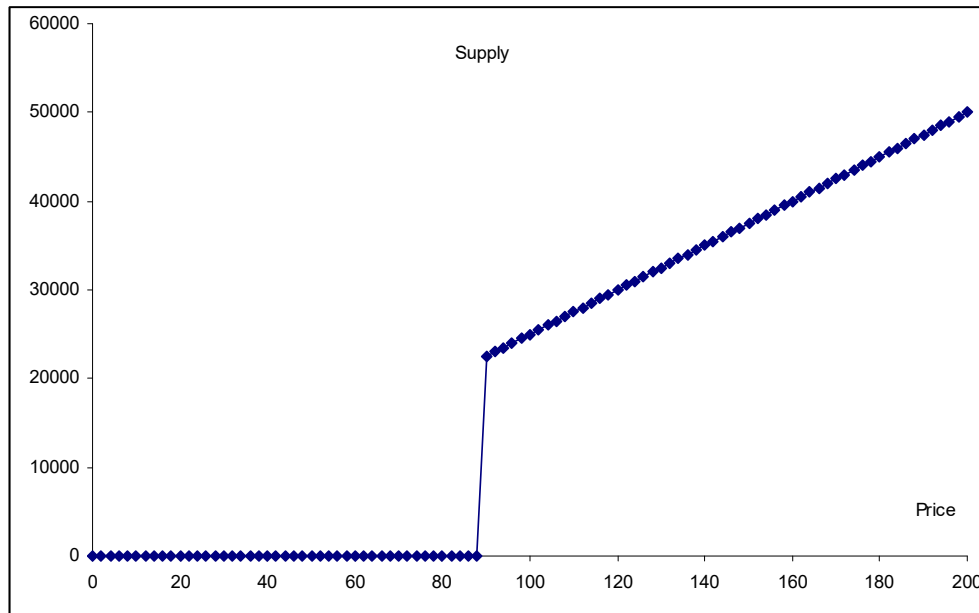
$$P > \sqrt{8,000} = 89.44.$$

Supply function

$$q = 250 P \text{ if } P > 89.44$$

$$q = 0 \text{ if } P < 89.44$$

Graphically, it is



That makes sense. As we have seen in c), at  $P=80$  the firm is better off by shutting down and thus producing zero. For large enough price, it's better to produce. Solution above shows that if  $P < 89.44$ , the firm is better off by shutting down and thus producing zero. If  $P > 89.44$ , the firm is better off by producing  $q=250 P$ . (If  $P=89.44$ , the firm is indifferent between shutting down and producing  $q=250 P$ .)

**2. The plant can produce two products, A and B. Fixed cost of running the plant is \$1,000,000 per month, irrespective of how much is produced; total cost of producing  $q_A$  units of product A is  $\$0.002 q_A^2$ , and total cost of producing  $q_B$  units of product B is  $\$0.002 q_B^2$ . The per-unit market price for product A is  $P_A=\$80$  and for product B it is  $P_B=\$60$ . (All costs and prices refer to long-run values.)**

**a) How many units of products A and B should the firm produce?**

Profit for A will be maximized if marginal revenue equals marginal cost:

$MC(q) = 0.004 q_A$ ,  $MR(q) = P_A = 80$ , so

$$q_A = 80 / 0.004 = 20,000$$

Corresponding profit from producing product A (without fixed costs)

$$80 \times 20,000 - 0.002 \times 20,000^2 = 800,000$$

Profit for B will be maximized if marginal revenue equals marginal cost:

$MC(q) = 0.004 q_B$ ,  $MR(q) = P_B = 60$ , so

$$q_B = 60 / 0.004 = 15,000$$

Corresponding Profit is

$$60 \times 15,000 - 0.002 \times 15,000^2 = 450,000$$

The total profit for the plant is:

$$800,000 + 450,000 - 1,000,000 = 250,000$$

**The accounting department splits the fixed cost (\$1,000,000 per month) of running the whole plant equally between the two products.**

**b) Is product A making a profit or a loss?**

For A:

$$800,000 - 500,000 = 300,000$$

It is a profit

**c) Is product B making a profit or a loss?**

For B:

$$450,000 - 500,000 = -50,000$$

It is a loss

**d) Should we stop producing the product that is making a loss according to the report of accounting department?**

No, this is a wrong decision: if the costs are split evenly, then B is making loss. If we shut it down, then the whole plant is making a loss. But originally (in question a)) the plant was making profit. So the message is that we should be careful about allocating costs and when thinking about shutting down particular product, we should take into account only the costs (either variable or fixed) that are associated with that product.