Solution Set 5

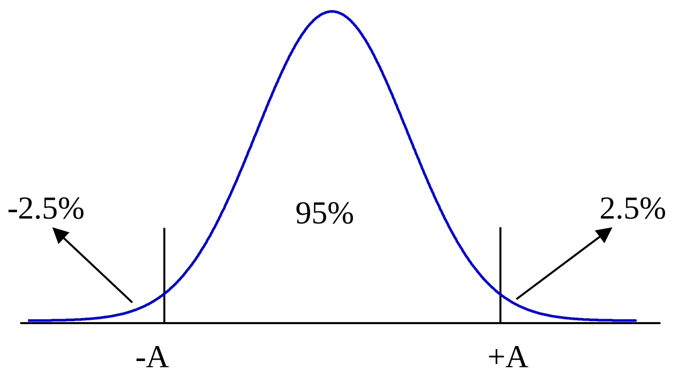
Normal distribution

1. No transformation required. We can use this statistic

a) Prob ($Z > 1.645$) = 0.05

b) Prob ($Z < -1.645$) = 0.05

c) Prob ($-A < Z < A$) = 0.95



Prob (Z > A) = 0.025 = F (Z)

Prob (Z < -A) = -0.025

The corresponding Z value is ± 1.96

2. Let x = age of a student

a) Prob ($x < 27$) = ?

$$Z = \frac{x-\mu}{\sigma} = \frac{27-28.7}{2.4} = -0.71$$

Prob ($x < 27$) = Prob ($Z < -0.71$) = 1 - 0.7611 = 0.2389

b) Prob ($x > 31$) = ?

$$Z = \frac{x-\mu}{\sigma} = \frac{31-28.7}{2.4} = 0.96$$

Prob ($x > 31$) = Prob ($Z > 0.96$) = 0.1685

3. a) Let x = weight of a baby

$$\text{Prob } (x > 4) = \text{Prob}\left(Z > \frac{x-\mu}{\sigma}\right)$$

$$= \text{Prob}\left(Z > \frac{4-3.25}{0.35}\right)$$

Prob ($x > 4$) = Prob ($Z > 2.14$) = 0.0162

= 1.62%

b) Prob (2.5 < *x* < 3.5)

$$Z_1 = \frac{2.5 - 3.25}{0.35} = -2.14$$

$$Z_2 = \frac{3.5 - 3.25}{0.35} = 0.71$$

Prob (2.5 < *x* < 3.5) = Prob (-2.14 < *Z* < 0.71)

$= F(Z_1) - F(Z_2) = 0.9838 - 0.2389$

= 0.7449 = 74.5%

c) Prob (*x* < *value*) = 0.22

Prob (*x* $\geq$ *value*) = 0.78 = *F*(*Z*)

$$Z = -0.77 = \frac{x - \mu}{\sigma}$$

$$x = \mu + Z * \sigma$$

= 3.25 – 0.77 $*$ 0.35

x = 2.98

4. Let *x* = monthly percentage changes in D-J

a) Prob (x > 0)

$$Z = \frac{x - \mu}{\sigma} = \frac{0 - 0.65}{3.5} = -0.186$$

Prob (*x* > 0) = Prob (*Z* > -0.186) = .57 = 57%

b) Prob (*x* > 5)

$$Z = \frac{x - \mu}{\sigma} = \frac{5 - .065}{3.5} = 1.24$$

Prob (*x* > 5) = Prob (*Z* > 1.24) = .107 = 10.7%

c) Prob (*x* > *value*) = 0.05

$$Z = 1.64$$

$$1.64 = \frac{x - .65}{3.5}$$

x = 1.64 $*$ 3.5 + .65 = 6.39

x = 6.39%

5. a) Prob(x>1272)

$$z = \frac{x-\mu}{\sigma} = \frac{1272-1200}{36} = 2$$

=Prob(z>2)=.0228=2.28%

b) Prob(*x*<1146)

$$z = \frac{x-\mu}{\sigma} = \frac{1146-1200}{36} = -1.5$$

=Prob(*Z*<1.5)=1-.9332=.0668=6.68%

c) Prob(x>value)=0.1

$$\Rightarrow z = 1.28 = \frac{x-1200}{36}$$

x=1200+36×1.28=1246

d) Prob(x<value)=.2 $\Leftrightarrow$ Prob(x>value)=.8

$$\Rightarrow z = -.84 = \frac{x-1200}{36}$$

$$x = 1200 - .84 * 36$$

$$x = 1170$$

6. *x* = travel time, follows a normal distribution with $\mu = 60$, $\sigma = 6$

Prob $(x > 70)$?

$$Z = \frac{70-60}{6} = 1.67$$

Prob $(Z > 1.67) = 0.0475$

Expected loss of revenue per passenger

$400€ * 0.0475 = 19€$ per passenger

7. Q follows a normal distribution with $\mu = 250$ $\sigma = ?$

Prob (Q > 275) = Prob $\left(Z > \frac{275-250}{\sigma}\right) = .15$

From the table, $Z = 1.04$

$$Z = \frac{275 - 250}{\sigma} = 1.04 \Rightarrow \sigma = 24.04$$

variance $\Rightarrow$ $\sigma^2 = 577.92$

8. *x* = consumption of laundry detergent, follows a normal distribution

99% of households use between 1 and 10 liters of detergent per week

Mean = $\frac{10-1}{2} = 5.5$

Prob$(1 < x < 10) = 0.99$

Prob$(x > 10) = 0.005$

$F(Z) = 0.005 \Rightarrow Z = 2.58$

$$Z = \frac{x-\mu}{\sigma} = \frac{10-5.5}{\sigma} = 2.58$$

$$\sigma = \frac{4.5}{2.58} = 1.74$$

9. *x* = time to go to school, follows a normal distribution with $\mu = 20$, $\sigma = 5$

Percentage of time she will be late = Prob$(x > 30)$

$$Z = \frac{x-\mu}{\sigma} = \frac{30-20}{5} = 2$$

Prob$(Z > 2) = 0.0228$

If she leaves her home 30 minutes the start of class, she will be late 2.3% of the time.

If she doesn't want to be late more than 1% of the time (for example)

Prob$(x > Time) = 0.01$

$F(Z) = 0.01 \Rightarrow Z = 2.33$

$$Z = \frac{x-\mu}{\sigma} = \frac{x-20}{5} = 2.33$$

$x = 20 + 5 * 2.33 = 31.65$

If she doesn't want to be late more than 1% of the time, she should leave home approximately 32 minutes before the start of class

10. *x* = height of men, follows a normal distribution with $\mu = 70$, $\sigma = 3$

Only 5% of men should be taller than $x = H$

Prob$(x > H) = 0.05$

$F(Z) = 0.05 \Rightarrow Z = 1.645$

$$Z = \frac{x-\mu}{\sigma} = \frac{H-70}{3} = 1.645$$

$H = 70 + 3 * 1.645 = 74.94$

We expect that 95% of the men will be shorter than 74.94 inches.
To have a 1-foot clearance, the doors should be:

$74.94 + 12 = 86.94$ inches high

11. x = number of ounces injected, follows a normal distribution with $\mu = 12$, $\sigma = 0.04$

Prob $(x < 11.9)$

$$Z = \frac{x-\mu}{\sigma} = \frac{11.9-12}{0.04} = -2.5$$

Prob $(Z < -2.5) = 1 - F(Z) = 1 - 0.9938$

$= 0.0062$

12. a) x = errors in the forecast, follows a normal distribution with $\mu = 0$, $\sigma = 10$

Forecasts inaccurate by more than 15% mean

$x > 15$ or $x < -15$

Prob $(x > 15)$ + Prob $(x < -15)$

As $\mu = 0$, these two probabilities are the same

$$Z = \frac{x-\mu}{\sigma} = \frac{15-0}{10} = 1.5$$

Prob $(Z > 1.5) = 0.0668$

Prob $(x > 15)$ + Prob $(x < -15) = 2 * 0.0668$

$= 0.1336$

b) Prob $(-c < x < +c) = 0.05$

$$\text{Prob}\,(x > +c) = \frac{1-0.05}{2} = 0.475$$

$F(Z) = 0.475 \Rightarrow Z = 0.06$

$$Z = \frac{x-\mu}{\sigma} = \frac{c-0}{10} = 0.06$$

$c = 0.06 * 10 = 0.6$

To earn the bonus, the analyst must give a forecast within 0.6% of the true value.