Solution Set 3

1. a) $R'(x) = 50 - 2x$; $R'(8) = 34$ $R(x)$ is increasing at $x = 8$

b) $C'(x) = 2x + 10$; $C'(8) = 26$ $C(x)$ is increasing at $x = 8$

c) $P'(x) = 6x - 28$; $P'(8) = 20$ $P(x)$ is increasing at $x = 8$

d) $C'(x) = 9x^2 - 42x + 11$; $C'(8) = 251$ $C(x)$ is increasing at $x = 8$

2. a) $R(Q) = P.Q = (Q^2 + 4Q + 9)Q = Q^3 + 4Q^2 + 9Q$

$R'(Q) = 3Q^2 + 8Q + 9$

$R'(5) = 3(5)^2 + 8(5) + 9 = 124$

b) $R(Q) = \left(\frac{1}{2}Q^2 + 3Q + 8\right)Q = \frac{1}{2}Q^3 + 3Q^2 + 8Q$

$R'(Q) = 1.5Q^2 + 6Q + 8$

$R'(5) = 1.5(5)^2 + 6(5) + 8 = 75.5$

c) $R(Q) = \left(\frac{1}{4}Q + 60\right)Q = \frac{1}{4}Q^2 + 60Q$

$R'(Q) = \frac{1}{2}Q + 60$

$R'(5) = \frac{1}{2}(5) + 60 = 62.5$

3. Differentiate each of the following:

a) $f(x) = 13$ $f'(x) = 0$

b) $f(x) = -27$ $f'(x) = 0$

c) $f(x) = 7x - 12$ $f'(x) = 7$

d) $f(x) = 25 - 6x$ $f'(x) = -6$

e) $f(x) = 9x^4$ $f'(x) = 9 * 4x^3 = 36x^3$

f) $f(x) = -5x^7$ $f'(x) = -5 * 7x^6 = -35x^6$

g) $f(x) = 4x^{-3}$ $f'(x) = 4 * (-3)x^{-4} = -12x^{-4}$

4. $T(x) = x^2 - 30x + 240 \quad x \geq 1$

$T'(x) = 2x - 30$

$T'(x) = 0 = 2x - 30$
$2x = 30$

$x = 15$ max. or min.

$T''(x) = 2$ is positive so it corresponds to a minimum. Fifteen staff members will minimize average waiting time.

5. Profit = (Demand)(Price-cost)

$P = f(p) = (50 - 4p)(p - 5)$

$f(p) = (50 - 4p)(p - 5)$

$= 50p - 4p^2 - 250 + 20p$

$= -4p^2 + 70p - 250$

$f'(p) = -4(2)p + 70$

$f'(p) = -8p + 70 = 0$

$8p = 70$

$p = \frac{70}{8} = 8.75$ is a critical point

$f''(p) = -8$ is negative so it corresponds to a maximum and profit is maximized at 8.75.

If there is an upper bound of 8, we cannot set the price at 8.75. Notice that $f'(p) = -8p + 70,$ and $f'(8) = -8(8) + 70 = -64 + 70 = 6$ is positive. This means that when price = $8 profits can still be improved if we increase the price, but since we can't, we should charge $8.

6. a.) $x = 6000 - 30p$

$30p = 6000 - x$

$p = \frac{6000}{30} - \frac{x}{30} = 200 - \frac{x}{30}$

b) Marginal cost $\frac{d}{dx}(72000 + 60x)$

$c'(x) = 60$

c) $R = p * x = \left(200 - \frac{x}{30}\right)x = 200x - \frac{x^2}{30}$

d) Marginal revenue

$R'(x) = 200 - \frac{2x}{30} = 200 - \frac{x}{15}$

$R'(1500) = 200 - \frac{1500}{15} = 100$

At a production level of 1500 items, revenue is increasing at a rate of $100 per item.

$R'(4500) = 200 - \frac{4500}{15} = -100$

At a production level of 4500 items, revenue is decreasing at a rate of $100 per item.

7. a) Set up the profit function : Profit = Revenue - Cost

$P(Q) = 600Q - 5Q^2 - (320 + 20Q) = -5Q^2 + 580Q - 320$

Find the critical values by solving $P'(Q) = 0$

$P'(Q) = -10Q + 580 = 0 \Rightarrow Q = 58$ critical value.

$P''(Q) = -10$ is negative so it corresponds to a maximum and profit is maximized at $Q = 58$

b) Set up the profit function :

$P(Q) = 1300Q - 4Q^2 - (2000 + 100Q) = -4Q^2 + 1200Q - 2000$

Find the critical values by solving $P'(Q) = 0$

$P'(Q) = -8Q + 1200 = 0 \Rightarrow Q = 150$ critical value

$P''(Q) = -8$ is negative so it corresponds to a maximum and profit is maximized at $Q = 150$

8. $P(Q) = A - B * Q$

$TR(Q) = (A - B * Q) * Q$

$TR(Q) = AQ - BQ^2$

$MR(Q) = A - 2BQ$

9. $Q = 120 - 2P \qquad TC(Q) = Q^2$

$P(Q) = 60 - \frac{Q}{2}$

Total Profit = Total Revenue – Total Cost

$= TR(Q) - TC(Q)$

Total Revenue $= \left(60 - \frac{Q}{2}\right) * Q$

$TR(Q) = 60Q - \frac{Q^2}{2}$

Total Profit $= 60Q - \frac{Q^2}{2} - Q^2$

$TP(Q) = 60Q - 1.5Q^2$

Maximum Profit is found by

$TP'(Q) = 0$

$60 - 3Q + 0$

$Q + 20$

$P = 60 - \frac{20}{2} + 50$

Corresponding Total Profit

$TP(Q) = 60Q - 1.5Q^2$

$TP(20) = 60 * 20 - 1.5 * 20^2 = 600$

10. a) To find the rate at which profits are changing after 3 years, take the derivative of the profit function.

$f'(x) = 3 + 2x$

$f'(3) = 3 + 2(3) = \$9$ millions/year

b) To predict the level of profits when $x = 3$, evaluate the profit function at $x = 3$.

$f(x) = 20 + 3x + x^2$

$f(3) = 20 + 3(3) + 3^2$

$= 20 + 9 + 9$

$= \$38$ millions

11. a)

$$P = 125 - Q^{1.5}$$

$$TR = P * Q + \left(125 - Q^{1.5}\right) * Q = 125Q - Q^{2.5}$$

$$MR = R' = 125 - 2.5Q^{1.5}$$

b)

$$TR(10) = 125 * 10 - 10^{2.5} = 933.77$$

$$MR(10) = 125 - 2.5 * 10^{1.5} = 45.94$$

At a level of production of Q = 10, the sale of one extra unit adds 45.94 to the total revenue.

c)

$$MR = 0$$

$$125 - 2.5Q^{1.5} = 0$$

$$Q^{1.5} = \frac{125}{2.5} = 50$$

$$\ln Q^{1.5} = \ln 50$$

$$1.5 \ln Q = \ln 50$$

$$\ln Q = \frac{\ln 50}{1.5} = \frac{3.912}{1.5} = 2.61$$

$$Q = e^{2.61} = 13.6$$

When $Q > 13.6$, $MR < 0$ and the total revenue starts to reduce.