Solution Set 2

1. a) $\left(\frac{x}{y}\right)^3\left(\frac{x}{z}\right)^{-2} = \left(\frac{x}{y}\right)^3\left(\frac{z}{x}\right)^2 = \frac{x^3z^2}{y^3x^2} = \frac{x^{3-2}z^2}{y^3} = \frac{xz^2}{y^3}$

b) $\frac{x^3y^2}{x^5y^{-2}} = \frac{x^3y^2y^2}{x^5} = x^{3-5}y^4 = x^{-2}y^4 = \frac{y^4}{x^2}$

c) $\frac{24x^5y^3z^7}{6x^3y^2z^4} = 4x^2yz^3$

d) $\frac{120xy^3z^7}{6x^3y^2z^4} = 20\frac{yz^3}{x^2}$

2. a) $(9^{3/2}) = (\sqrt{9})^3 = 3^3 = 27$

b) $(8)^{4/3} = (8^{1/3})^4 = \left(\sqrt[3]{8}\right)^4 = 2^4 = 16$

c) $\left(\frac{1}{4}\right)^{\frac{5}{2}} = \left[\left(\frac{1}{4}\right)^{\frac{1}{2}}\right]^5 = \left(\frac{1}{2}\right)^5 = \frac{1^5}{2^5} = \frac{1}{32}$

d) $27^{-2/3} = \frac{1}{(27)^{2/3}} = \frac{1}{\left[(27)^{1/3}\right]^2} = \frac{1}{\left[\sqrt[3]{27}\right]^2} = \frac{1}{3^2} = \frac{1}{9}$

e) $(-4)^5 = (-4)*(-4)*(-4)*(-4)*(-4) = -1024$

f) $(-2)^6 = +64$

3. a) $A = P(1+r)^t$

$= \$2000(1+.05)^{10} = \$3258$

b) $A = P\left(1+\frac{r}{n}\right)^{nt}$

$= 1\,500\,000(1+\frac{.07}{2})^{2x6}$

$= 1\,500\,000(1+.035)^{12}$

$= \$2266603$

4. Interest = (principal)(rate)(time)
= ($800)(.04)(4)
=$128

You owe = amount borrowed + interest
= $800+$128=$928

5. $S = P(1+\frac{r}{p})^{pt}$

$$P = \frac{S}{(1+\frac{r}{p})^{pt}}$$

$$P = S(1+\frac{r}{p})^{-pt} =$$

$$16\ 436.2(1+\frac{0.05}{4})^{-4.10} = 10\ 000$$

6. $A = P(1+r)^t$

$$P + 300 = P(1+0.03)^2$$

$$300 = P(1.03)^2 - P$$

$$300 = P(1.03^2 - 1)$$

$$P = \frac{300}{(1.03)^2 - 1} = \frac{300}{0.0609} = 4926.11$$

7. Where possible, evaluate or simplify using the laws of logarithms.

a) $\log_a(2xy)$

$= \log_a 2 + \log_a x + \log_a y$ (product rule)

b) $\log_a(\frac{x^2y^3}{z^4}) = \log_a x^2y^3 - \log_a z^4$ (quotient rule)

$= \log_a x^2 + \log_a y^3 - \log_a z^4$ (product rule)

$= 2\log_a x + 3\log_a y - 4\log_a z$ (power rule)

c) $\log_a(zx + y)$ cannot be further simplified

d) $\log_a(x^a) = a\log_a x$

e) $\log_a(\frac{a^2x^3}{3}) = \log_a(a^2x^3) - \log_a 3$ (quotient rule)

$= \log_a a^2 + \log_a x^3 - \log_a 3$ (product rule)

$= 2\log_a a + 3\log_a x - \log_a 3$ (power rule)

$= 2 + 3\log_a x - \log_a 3$ ($\log_a a = 1$)

f) $\log_b \left[P(1+r)^t\right] = \log_b P + \log_b (1+r)^t$

$= \log_b P + t\log_b (1+r)$

g) $\ln(100e^{-0.01t}) = \ln 100 + \ln(e^{-0.01t}) = \ln 10^2 + \ln\left(e^{-0.01t}\right)$

$= 2\ln 10 - 0.01t = 2 * 2.30 - 0.01t = 4.6 - 0.01t$

h) $\log_{10}(67 * 10^{-0.12x}) = \log_{10} 67 + \log_{10} 10^{-0.12x}$

$= 1.826 - 0.12x$

8. **a)** $(.58)^x = 5.67$

$\ln(.58)^x = \ln(5.67)$

$x\ln(.58) = \ln(5.67)$

$x = \frac{\ln(5.67)}{\ln(.58)} = \frac{1.735}{-0.545} = -3.185$

b) $e^{3x} = 403.43$

$\ln(e^{3x}) = \ln(403.43)$

$3x = 6$

$x = \frac{6}{3} = 2$

c) $4\ln x - 10 = 0$

$4\ln x = 10$

$\ln x = 2.5$

$x = e^{2.5}$

$x = 12.18$

d) $3e^{x-4} = 24$

$e^{x-4} = 8$

$x - 4 = \ln 8$

$\boldsymbol{x = 6.08}$

e) $\ln(x+6) - \ln(x-3) = 1$

$$\ln\frac{x+6}{x-3} = 1$$

$$e^{\ln\frac{x+6}{x-3}} = e^1$$

$$\frac{x+6}{x-3} = 2.7$$

$$x+6 = 2.7(x-3)$$

$$x+6 = 2.7x - 8.1$$

$$6+8.1 = 2.7x - x$$

$$14.1 = 1.7x$$

$$x = \frac{14.1}{1.7} = 8.29$$

9. The exponential growth can be written as

$$GDP \times e^{rt}$$

So, after 7 years

$$GDP \times e^{r*7} = 2GDP$$

$$e^{7r} = 2$$

$$\ln\left(e^{7r}\right) = \ln 2$$

$$7r = 0.693$$

$$r = \frac{0.693}{7}$$

$$r = 0.099 = 9.9\%$$

10. $S(t) = e^{2t-0.2t^2}$ for $0 < t < 5$

$$20 = e^{2t-0.2t^2}$$

$$\ln 20 = 2t - 0.2t^2$$

$$2.9957 = 2t - 0.2t^2$$

$$0.2t^2 - 2t + 2.9957 = 0$$

a = 0.2
b = -2
c = 2.9957

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(0.2)(2.9957)}}{2(0.2)}$$

$$t = \frac{2 \pm \sqrt{4 - 2.3966}}{0.4} = \frac{2 \pm \sqrt{1.6034}}{0.4} = \frac{2 \pm 1.2663}{0.4}$$

$$t = \frac{2 + 1.2663}{0.4} = 8.17 \text{ not in domain}$$

$$t = \frac{2 - 1.2663}{0.4} = 1.83 \text{ weeks}$$

11. $A = P(1+r)^t$

$2P = P(1+.20)^t$

$2 = 1.2^t$

$\ln 2 = t \ln 1.2$

$$t = \frac{\ln 2}{\ln 1.2} = \frac{0.693}{0.182} = 3.8$$

it will take almost 4 years.

12. If money triples $A = 3P = Pe^{rt} = Pe^{0.1t}$

$3 = e^{0.1t}$

Applying ln to both sides,

$\ln 3 = 0.1t$

$1.09861 = 0.1t$

$t = 10.99$ years

it will take 11 years.

13. Let $t = 0$ for the base year 1985, then $t = 5$ for 1990.
Expressing the two sets of data points in terms of

$M = M_0 e^{rt}$, and recalling that $e^0 = 1$,

$$3.64 = M_0 e^{r(0)} = M_0$$

$$5.82 = M_0 e^{r(5)}$$

Substitute M_0 simplify algebraically,

$$5.82 = 3.64e^{5r}$$

$$\frac{5.82}{3.64} = 1.60 = e^{5r}$$

Then take the natural logarithm of each side and use a calculator.

$$\ln 1.60 = \ln e^{5r} = 5r$$

$$0.47 = 5r$$

$$r = \frac{0.47}{5} = 0.094$$

$$r = 9.4\%$$

Now place the values of M_0 and r in the desired form $M_0 e^{rt}$: $M = 3.64e^{.094t}$